Potential of Inverse Methods in Studies of Dissolved/Particulate TEI Exchanges

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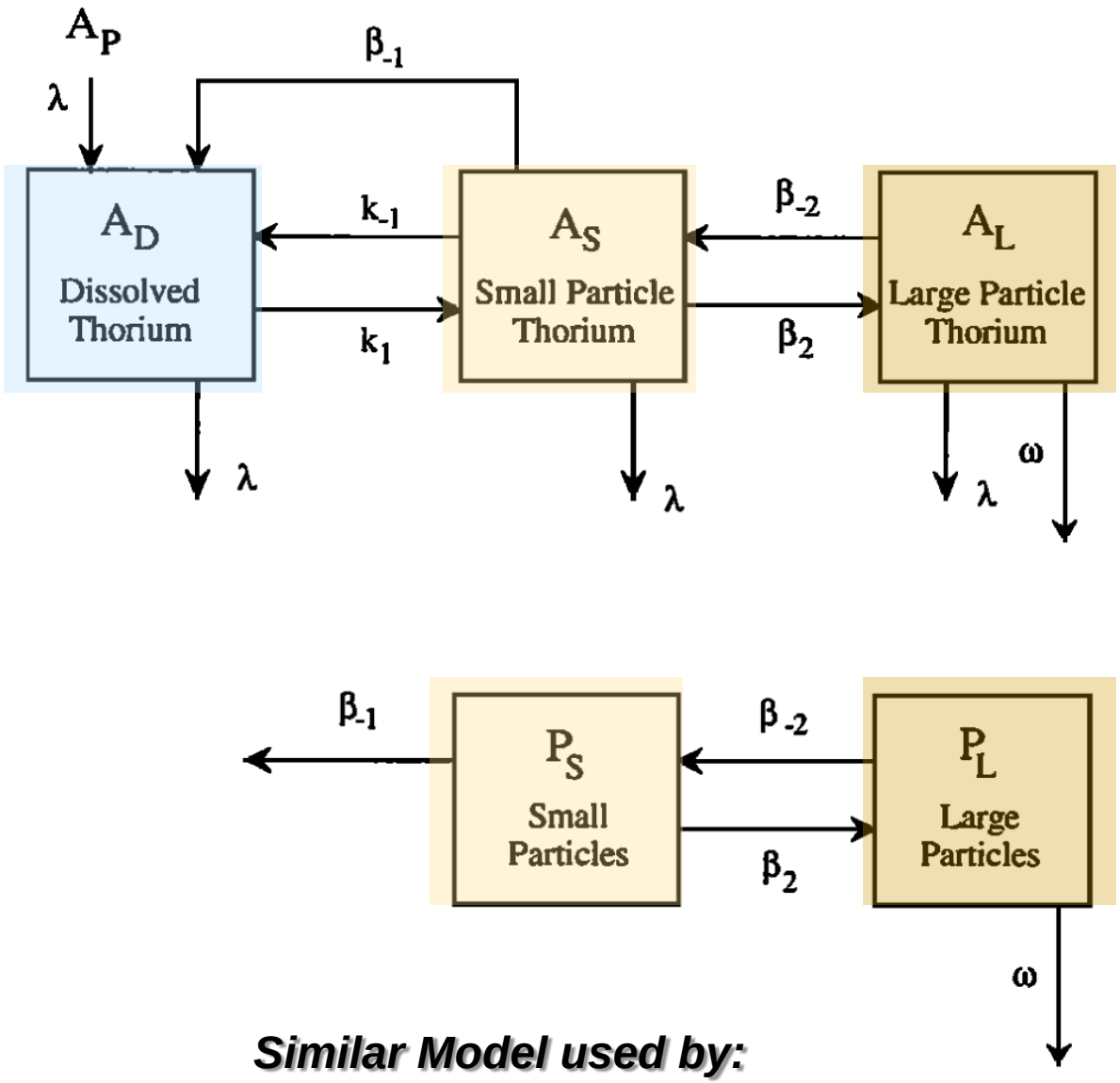


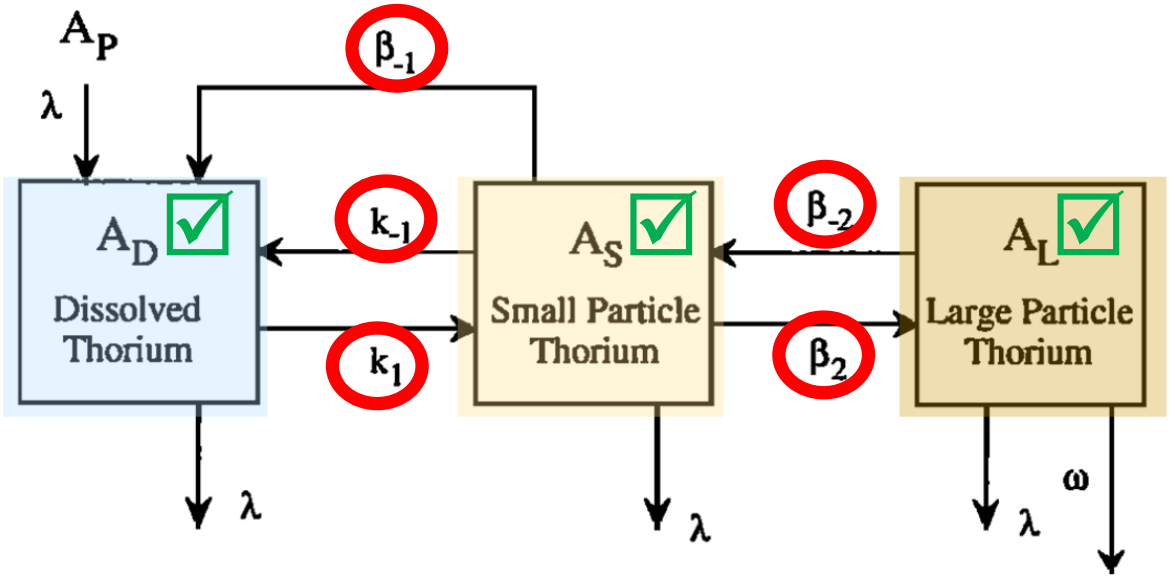
Similar Model used by:

Bacon et al, 1985
Murnane et al, 1990
Marchal+Lam, 2011

Ruiz-Pino, 1994
Athias et al, 2000

Questions:

- ***Can we estimate the rate constants k, β and w from data?***
- ***What type of data is necessary?***
- ***How much data do we need?***



^{228}Th
^{230}Th
^{234}Th

Steady-state budgets for A_D, A_S, and A_L

$$A_D: \quad 0 = S + A_s(k_{-1} + \beta_{-1}) - A_d(k_1 + \lambda) + T_{A_d}$$

$$A_S: \quad 0 = A_d k_1 + A'_L P_L \beta_{-2} - A_s(k_{-1} + \beta_{-1} + \beta_2 + \lambda) + T_{A_s}$$

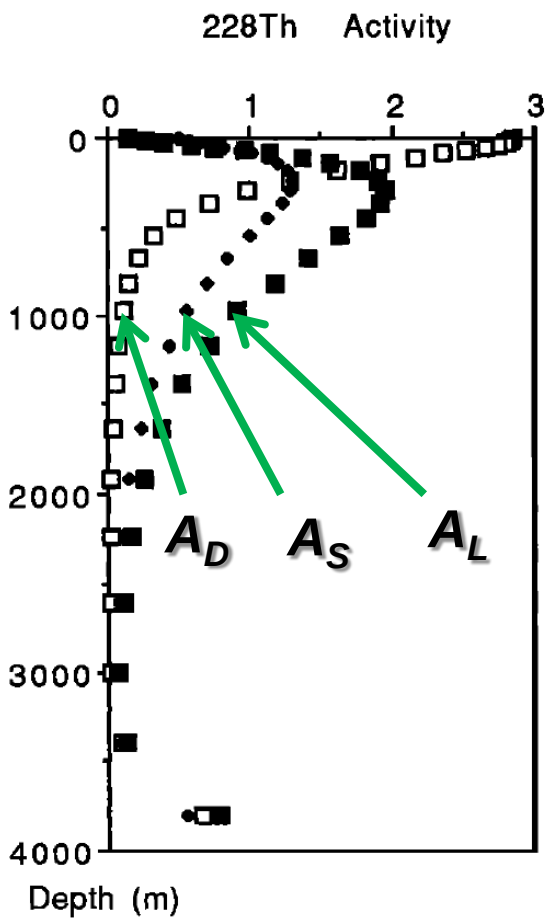
$$A_L: \quad 0 = A_s \beta_2 - A'_L P_L(\beta_{-2} + \lambda) - \frac{\partial F_{A_L}}{\partial z} + T_{A_L}$$

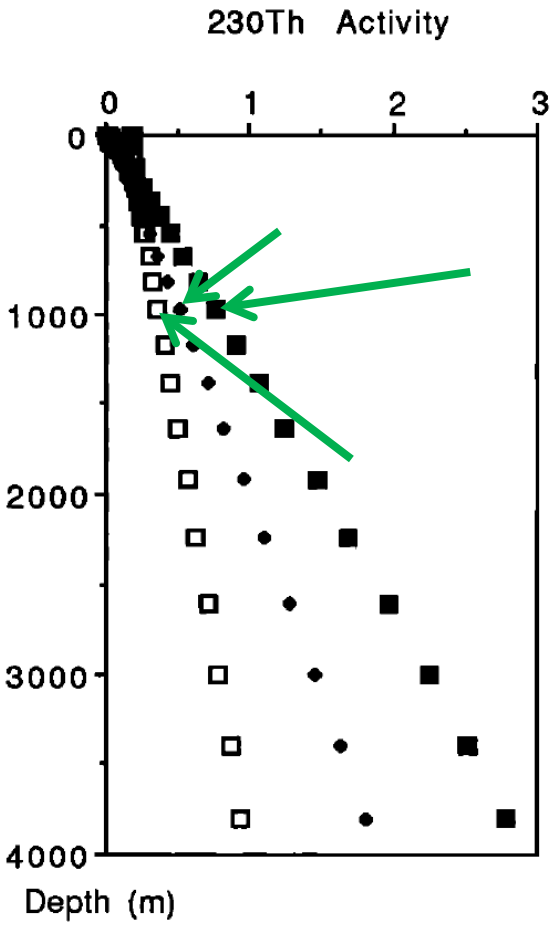
Murnane et al, 1990

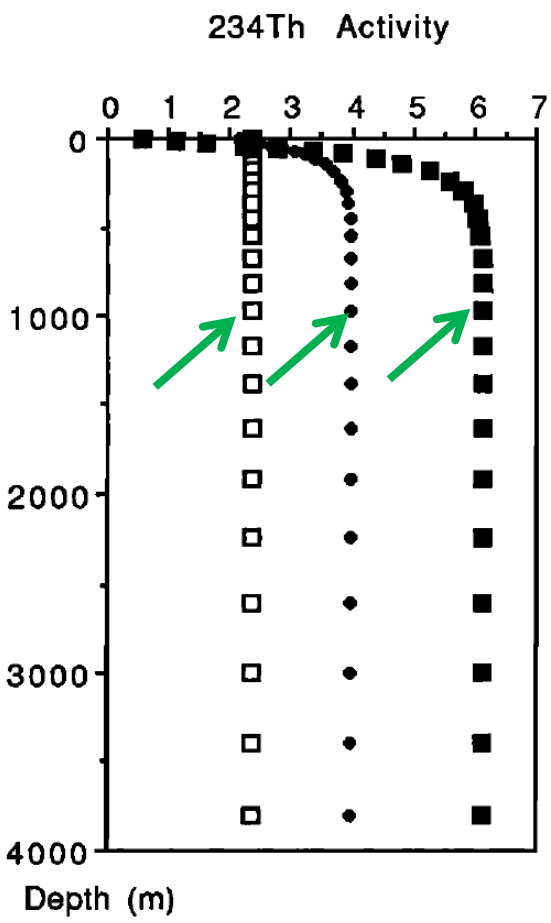
Matrix Notation:

$$
\begin{array}{l}
\\
^{228}Th \\
\\
\\
\\
^{230}Th \\
\\
\\
\\
^{234}Th \\
\\
\end{array}
\begin{bmatrix}
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L \\
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L \\
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L
\end{bmatrix}
\begin{bmatrix}
k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2}
\end{bmatrix}
=
\begin{bmatrix}
\\ \\ \\ \\ \\ \\ \\ \\ \\
\end{bmatrix}
$$

Modeled values







Murnane et al, 1990

$$
\begin{array}{l}
\\
^{228}\mathrm{Th} \\
\\
\\
\\
^{230}\mathrm{Th} \\
\\
\\
\\
^{234}\mathrm{Th} \\
\\
\end{array}
\begin{bmatrix}
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L \\
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L \\
-A_D & A_S & A_S & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L \\
0 & 0 & 0 & A_S & -A_L
\end{bmatrix}
\begin{bmatrix}
k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2}
\end{bmatrix}
=
\begin{bmatrix}
\\ \\ \\ \\ \\ \\ \\ \\ \\
\end{bmatrix}
$$

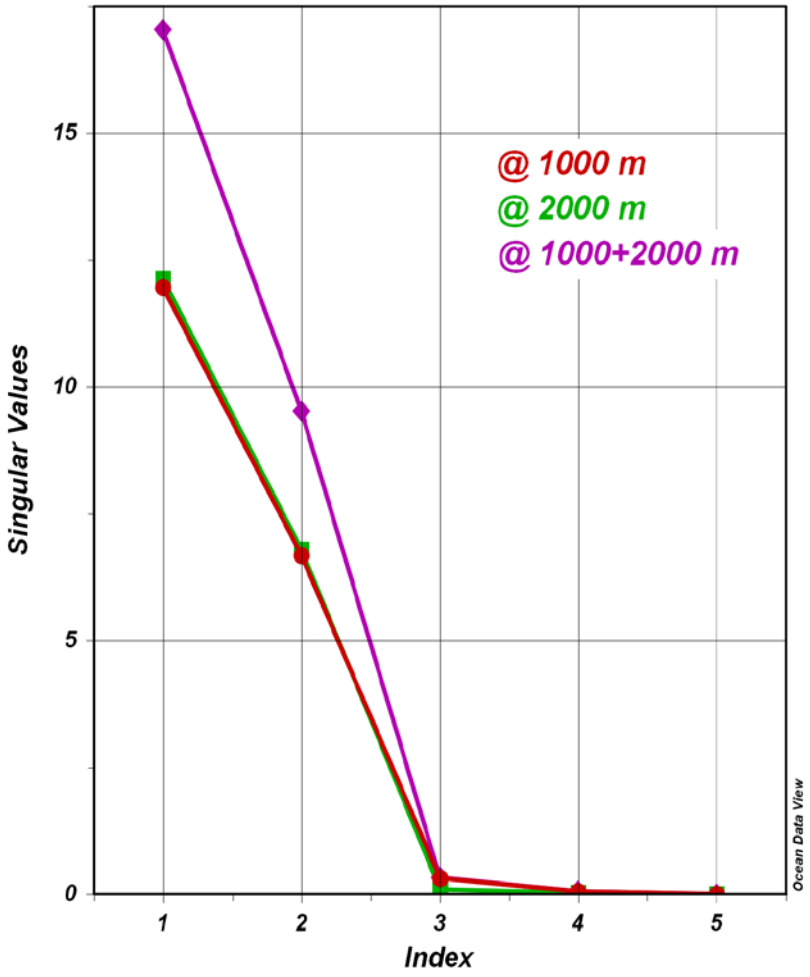
9 equations in 5 unknowns!

Overdetermined?

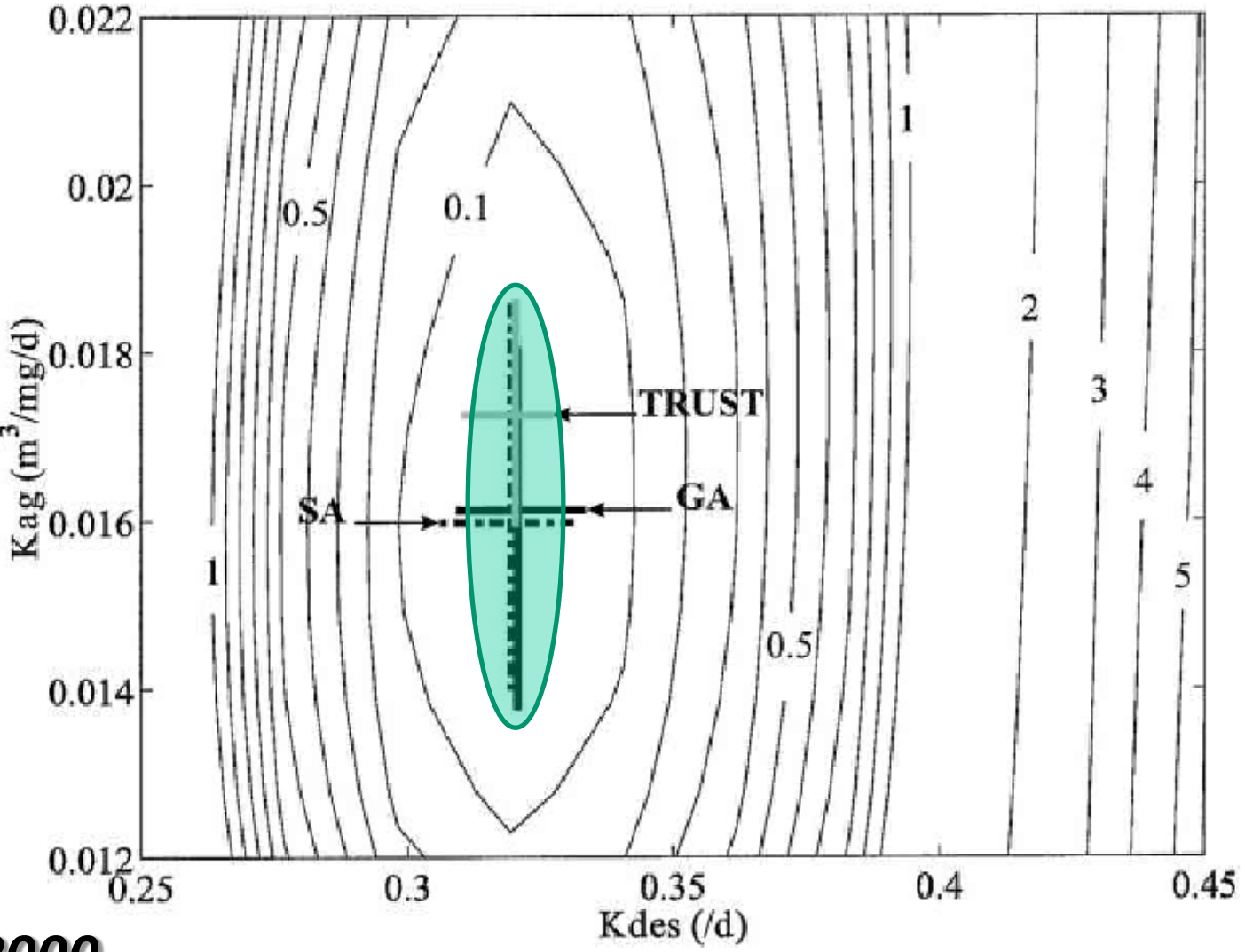
Perform Singular Value Decomposition of Matrix

Surprise:

- Only 2 independent equations
- System strongly underdetermined
- Same at other depths (e.g., 2000 m)
- Same when combining more than one depth (e.g., 1000 and 2000 m)
- Solutions can still be obtained, but is deficient and cannot fully resolve all unknowns.



Underdetermined systems have an infinite number of solutions:



Athias et al, 2000

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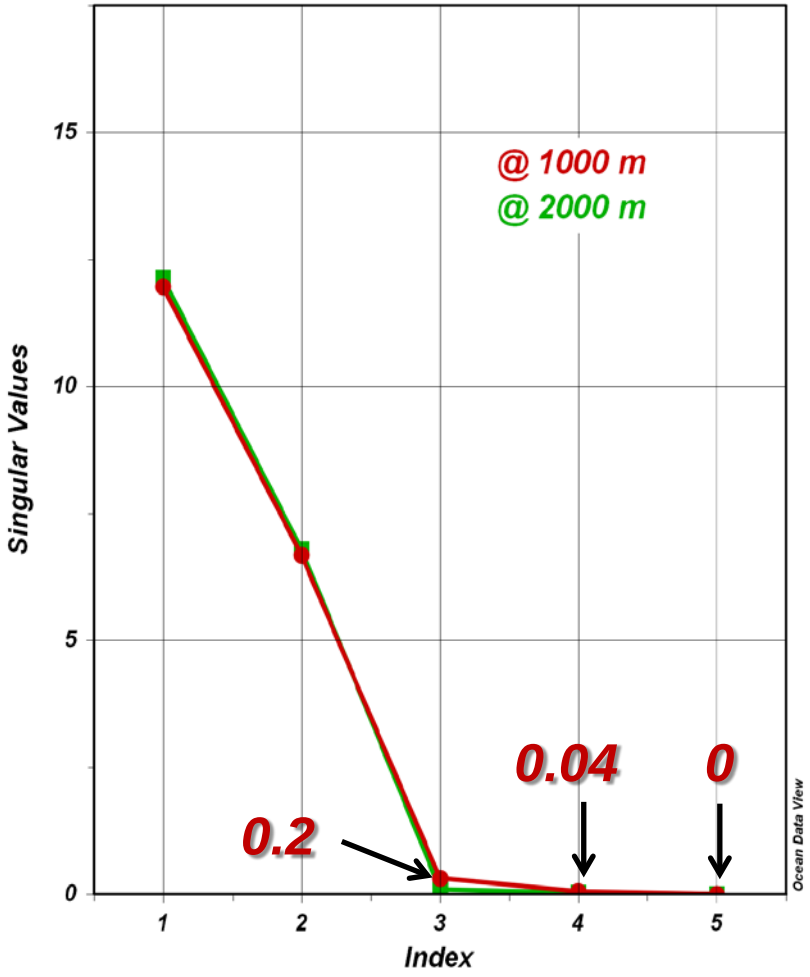


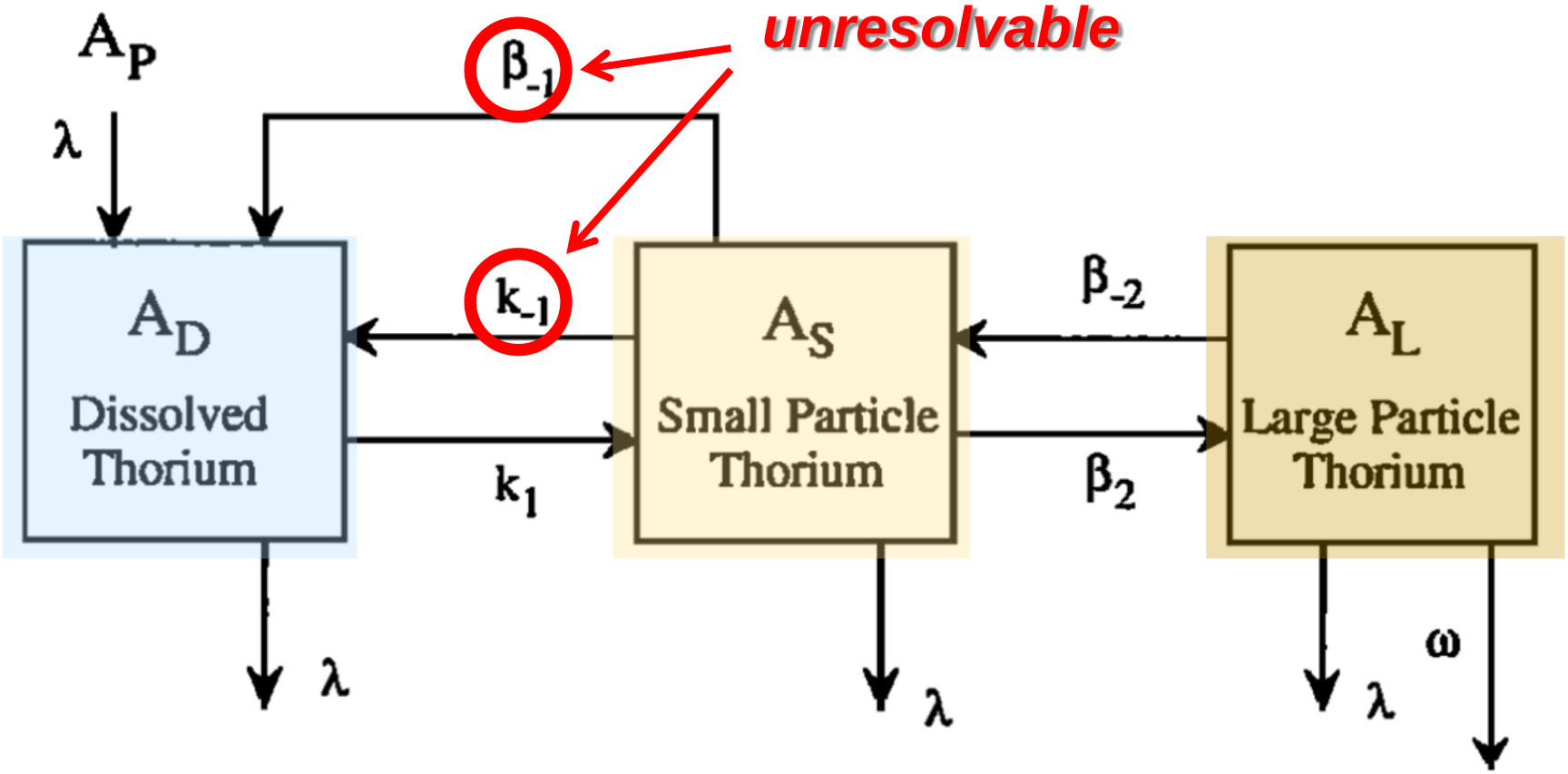
Resolution Matrix:

p=4; cond=300

@1000 m

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0.5 & 0.5 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix} \begin{Bmatrix} k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2} \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{Bmatrix}$$

- k_1, β_2 , and β_{-2} fully resolved, but
- $x_2 = (k_{-1} + \beta_{-1})/2$
- $x_3 = (k_{-1} + \beta_{-1})/2$
- cannot resolve k_{-1} and β_{-1}
- large condition number, e.g., solution very sensitive to data errors

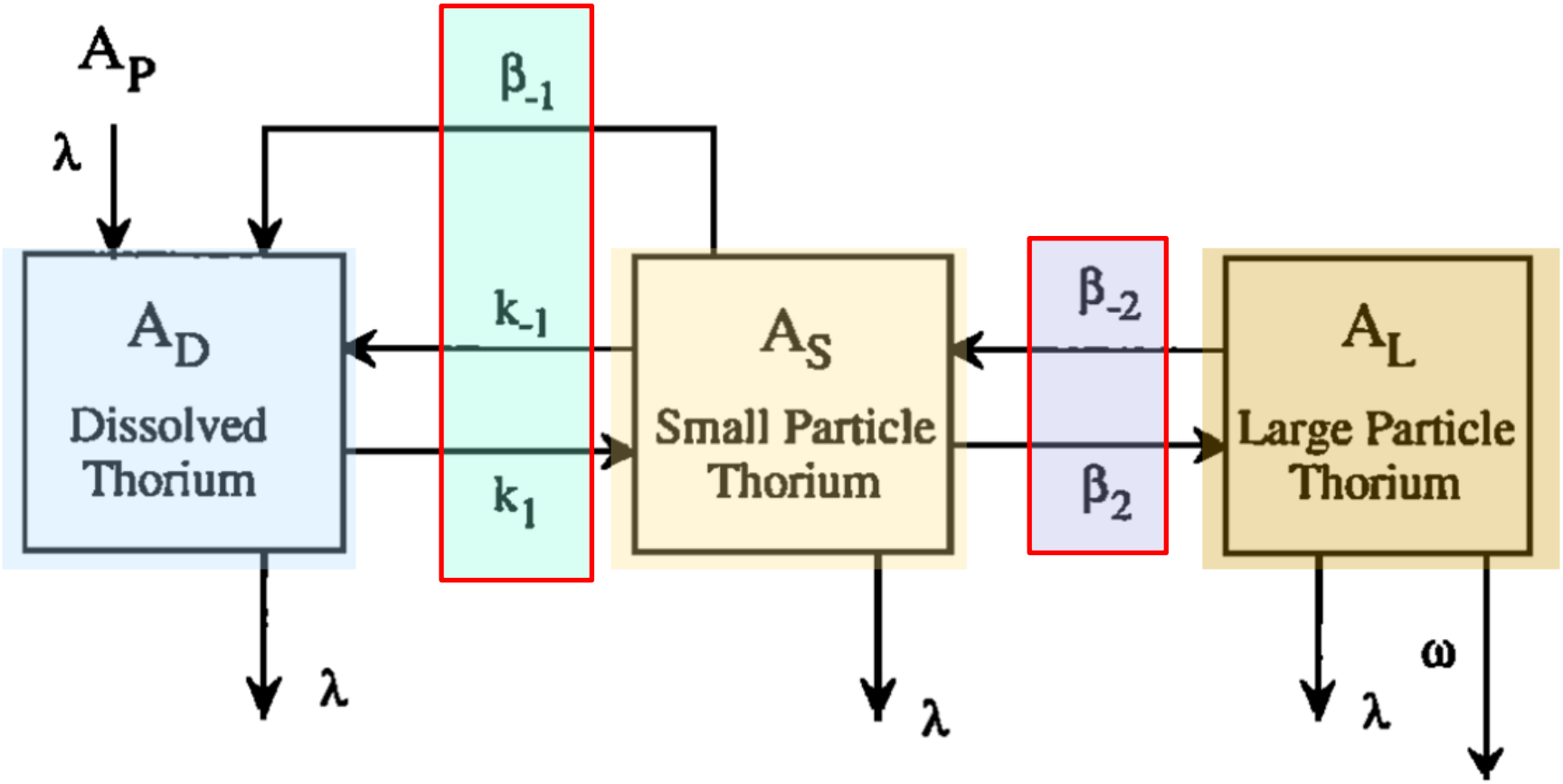

unresolvable
A_P
λ
β_{-1}
k_{-1}
A_D
Dissolved Thorium
k_1
A_S
Small Particle Thorium
β_{-2}
β_2
A_L
Large Particle Thorium
λ
λ
λ
ω

Resolution Matrix:

p=2; cond=1.8

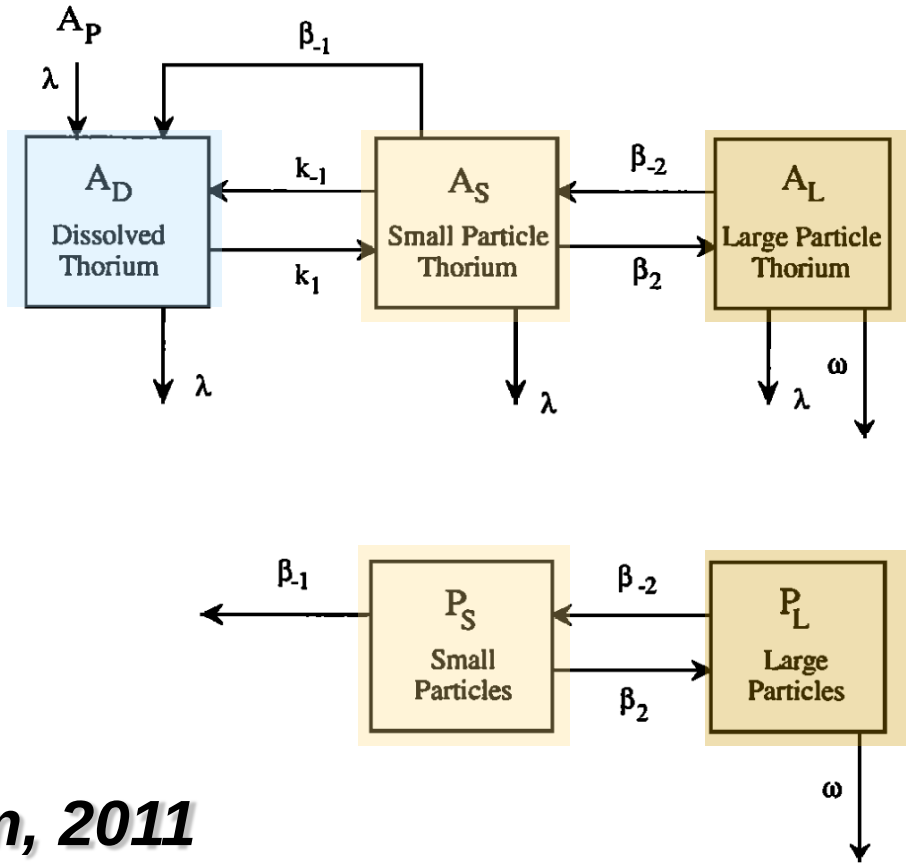
@1000 m

$$\begin{bmatrix} 0.15 & -0.25 & -0.25 & 0 & 0 \\ -0.25 & 0.43 & 0.43 & 0 & 0 \\ -0.25 & 0.43 & 0.43 & 0 & 0 \\ 0 & 0 & 0 & 0.29 & 0.46 \\ 0 & 0 & 0 & 0.46 & 0.71 \end{bmatrix} \begin{bmatrix} k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2} \end{bmatrix} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix}$$


A_P
λ
A_D
Dissolved Thorium
A_S
Small Particle Thorium
A_L
Large Particle Thorium
β_{-1}
k_{-1}
k_1
β_{-2}
β_2
λ
λ
λ
ω

Improvements:

- ***More A_D, A_S, and A_L data at additional depths do not help***
- ***Particle concentration data together with budgets for particle mass provide indepent information on β_{-1}, β_2, and β_{-2} and will help to resolve k's and β's.***



Marchal+Lam, 2011

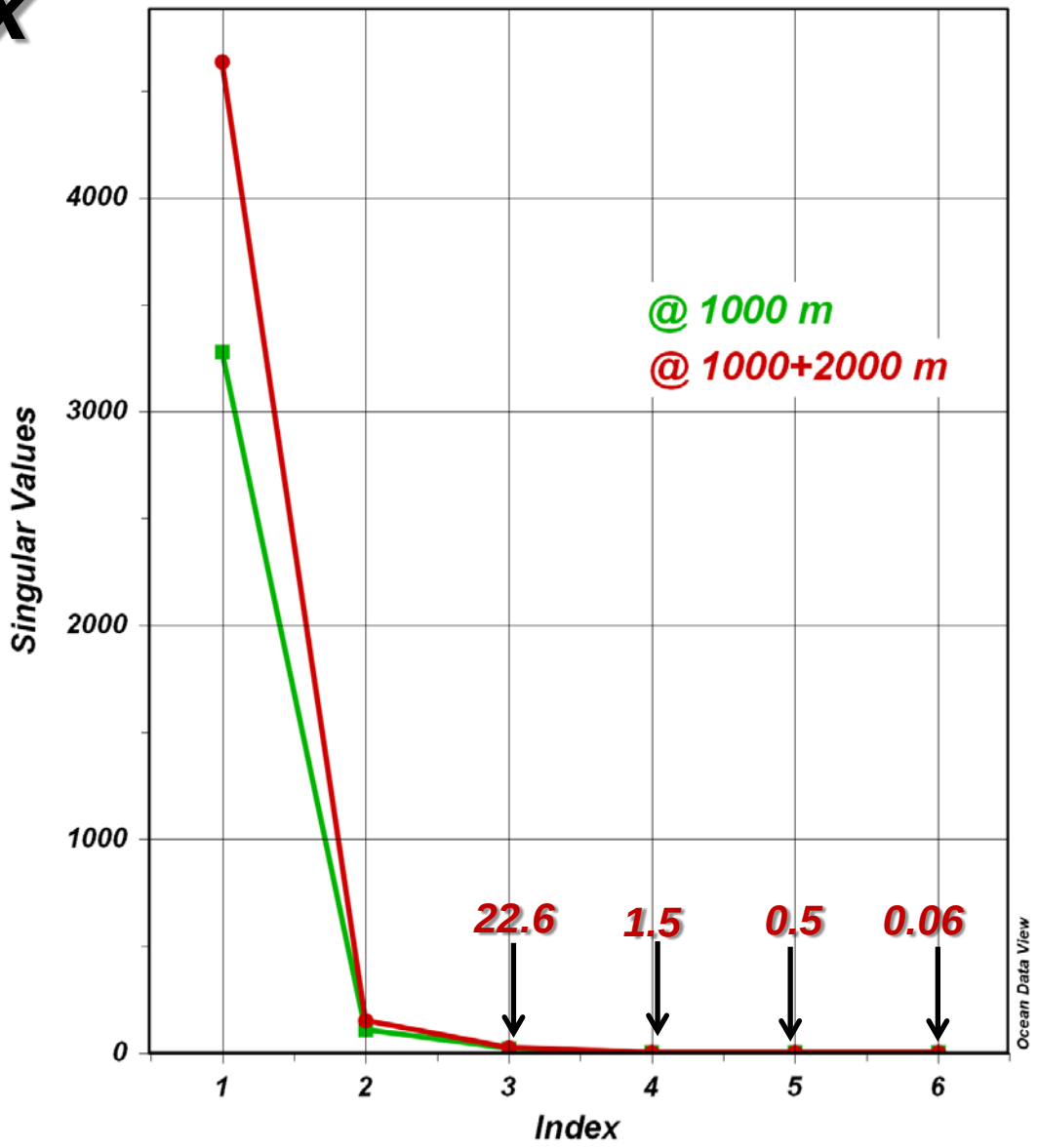
$$
\begin{bmatrix}
-A_D & A_S & A_S & 0 & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L & 0 \\
0 & 0 & 0 & A_S & -A_L & -(A_L)_z \\
-A_D & A_S & A_S & 0 & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L & 0 \\
0 & 0 & 0 & A_S & -A_L & -(A_L)_z \\
-A_D & A_S & A_S & 0 & 0 & 0 \\
A_D & -A_S & -A_S & -A_S & A_L & 0 \\
0 & 0 & 0 & A_S & -A_L & -(A_L)_z \\
0 & 0 & -P_S & -P_S & P_L & 0 \\
0 & 0 & 0 & P_S & -P_L & -(P_L)_z
\end{bmatrix}
\begin{bmatrix} k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2} \\ w \end{bmatrix}
=
\begin{bmatrix} \\ \\ \\ \\ \\ \\ \\ \\ \\ \\ \end{bmatrix}
$$

Particle Budgets

11 equations in 6 unknowns!

Overdetermined?

Perform Singular Value Decomposition of Matrix



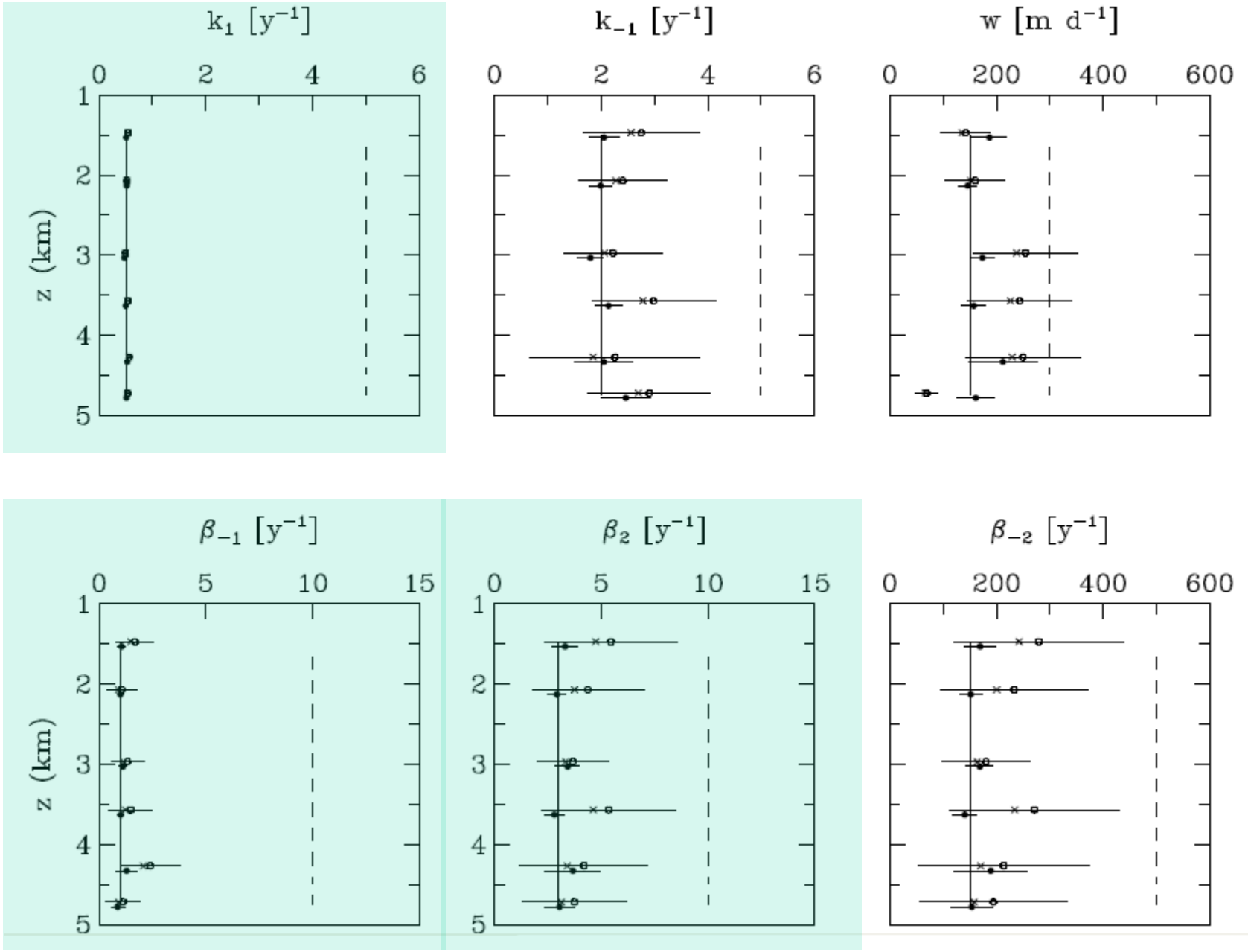
Resolution Matrix:

p=3; cond=163

@1000+2000 m

$$\begin{Bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{Bmatrix} \begin{Bmatrix} k_1 \\ k_{-1} \\ \beta_{-1} \\ \beta_2 \\ \beta_{-2} \\ w \end{Bmatrix} = \begin{Bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{Bmatrix}$$

- k_1, β_{-1} , and β_2 fully resolved, but
- no information about k_{-1}, β_{-2} , and w



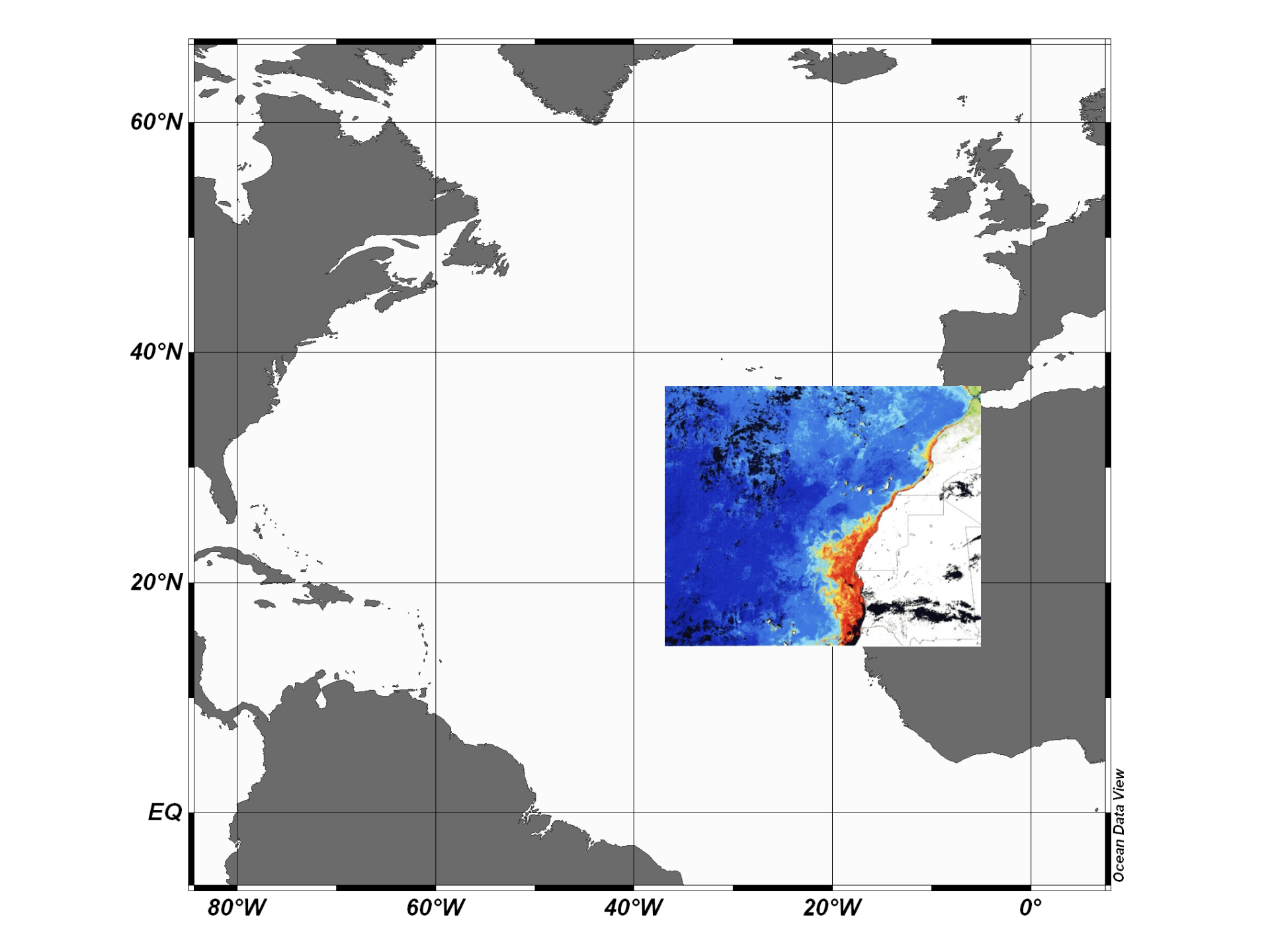
Marchal+Lam, 2011

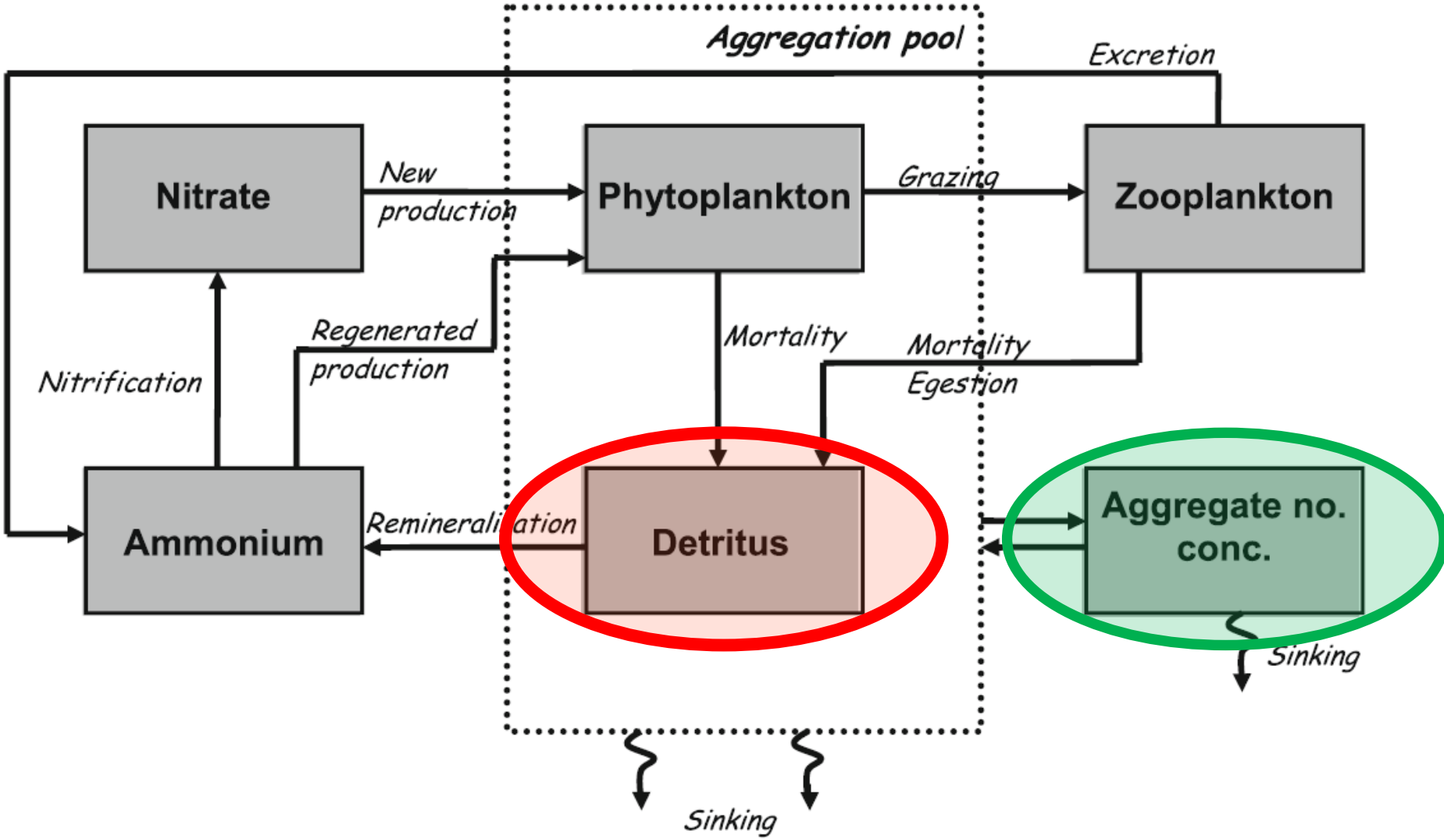
Conclusions: Part 1

- ***SVD allows analysis of linear systems: rank, condition, resolution, data resolution***
- ***Should always be done before solutions are calculated.***
- ***Be prepared for surprises: systems are often underdetermined***
- ***Combination of TEI and particle concentration data improved rank and resolution.***
- ***Are particle data available on a regular basis?***

Distributions of Biogenic and Lithogenic Particles

Modelers View


60°N
40°N
20°N
EQ
80°W
60°W
40°W
20°W
0°
Ocean Data View


Aggregation pool
Excretion
Nitrate
New production
Phytoplankton
Grazing
Zooplankton
Regenerated production
Mortality
Mortality
Egestion
Nitrification
Ammonium
Remineralization
Detritus
Aggregate no. conc.
Sinking
Sinking

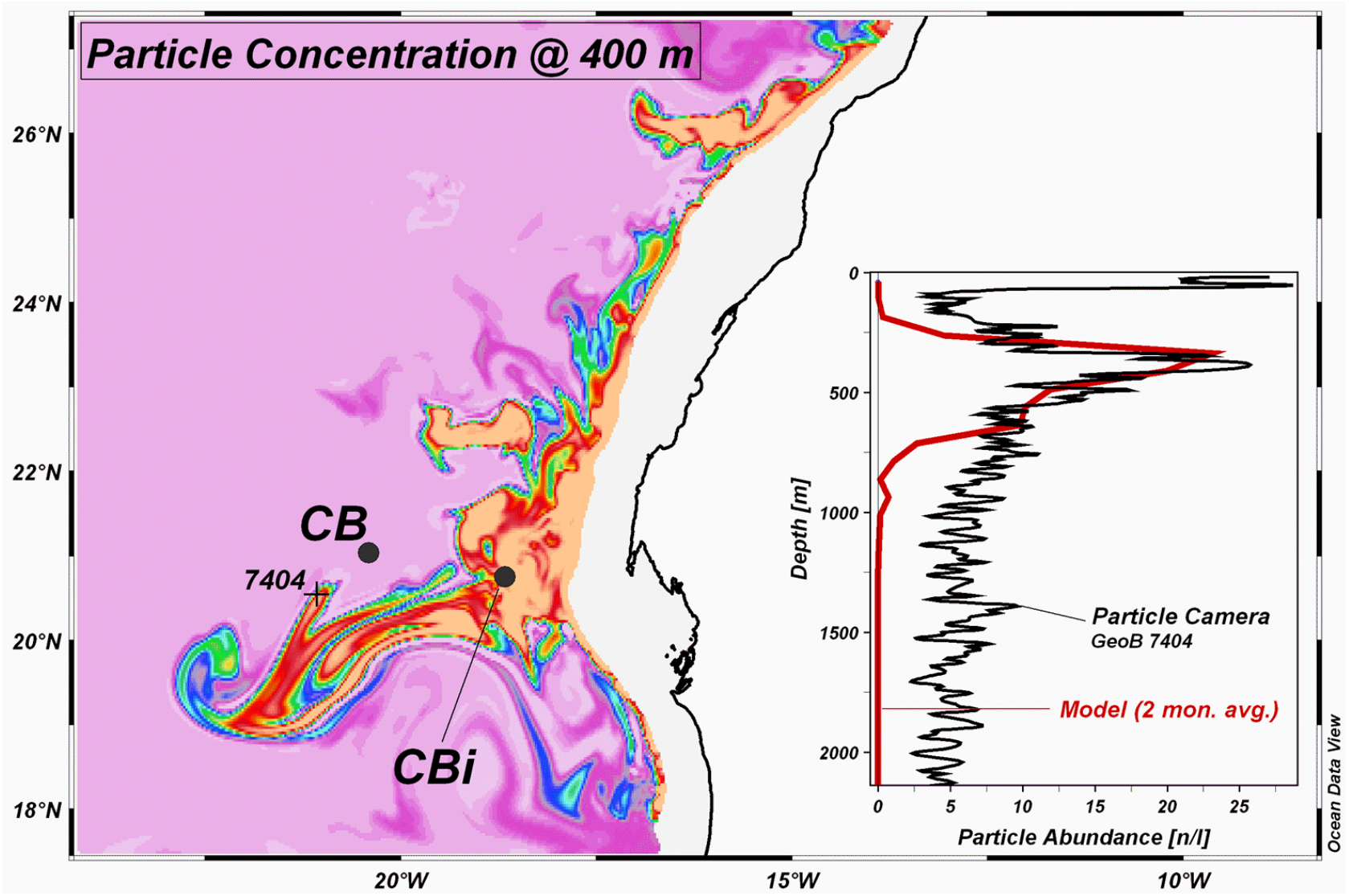
Animation: Large detritus map

Karakas, unpublished

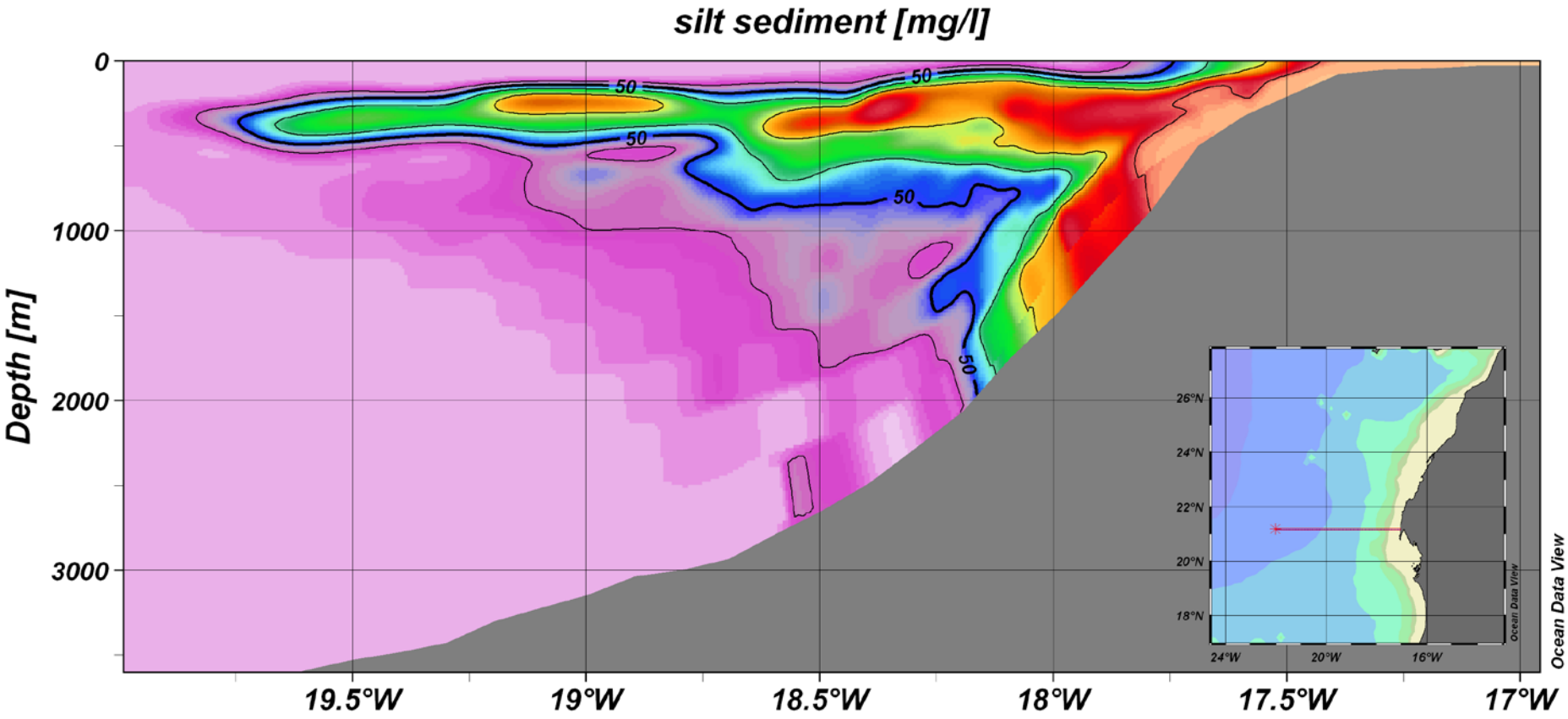
Animation: Large detritus section

Karakas, unpublished

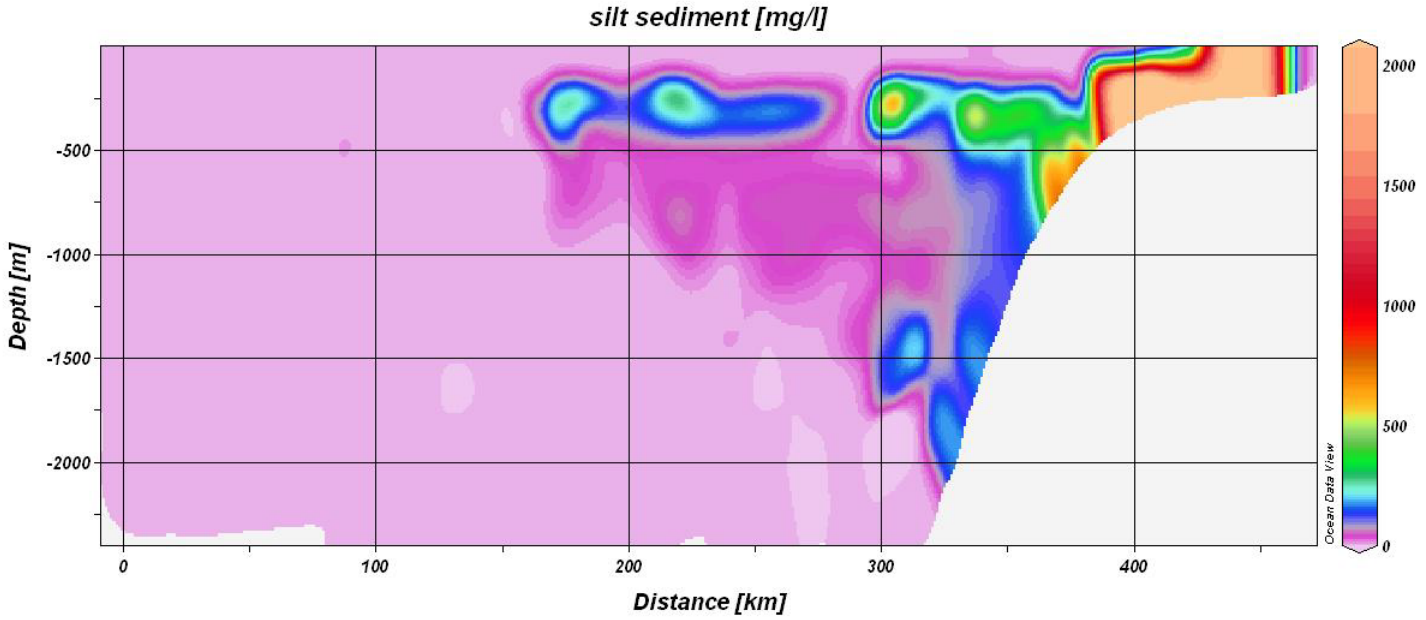
Sediment Resuspension – Nepheloid Layers



Karakas etal, 1996



Karakas etal, 1996



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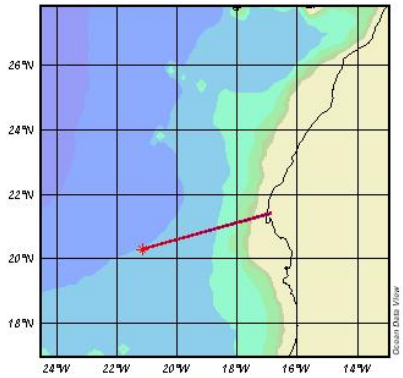

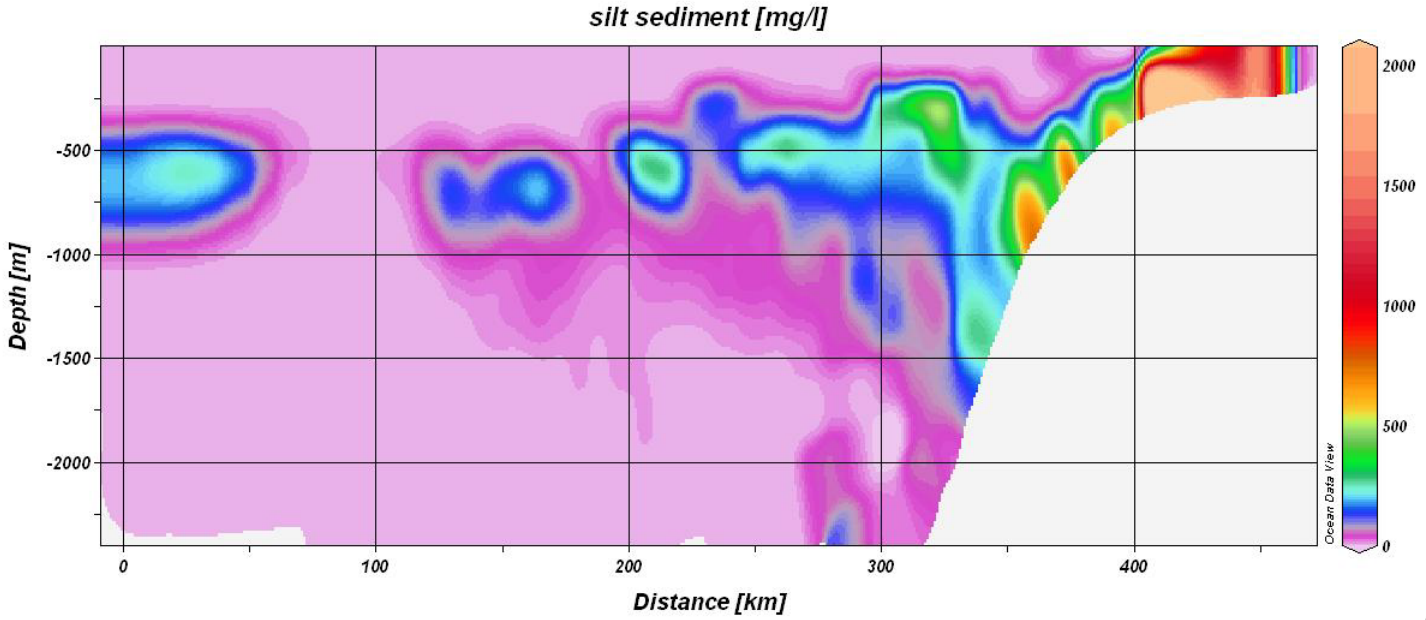

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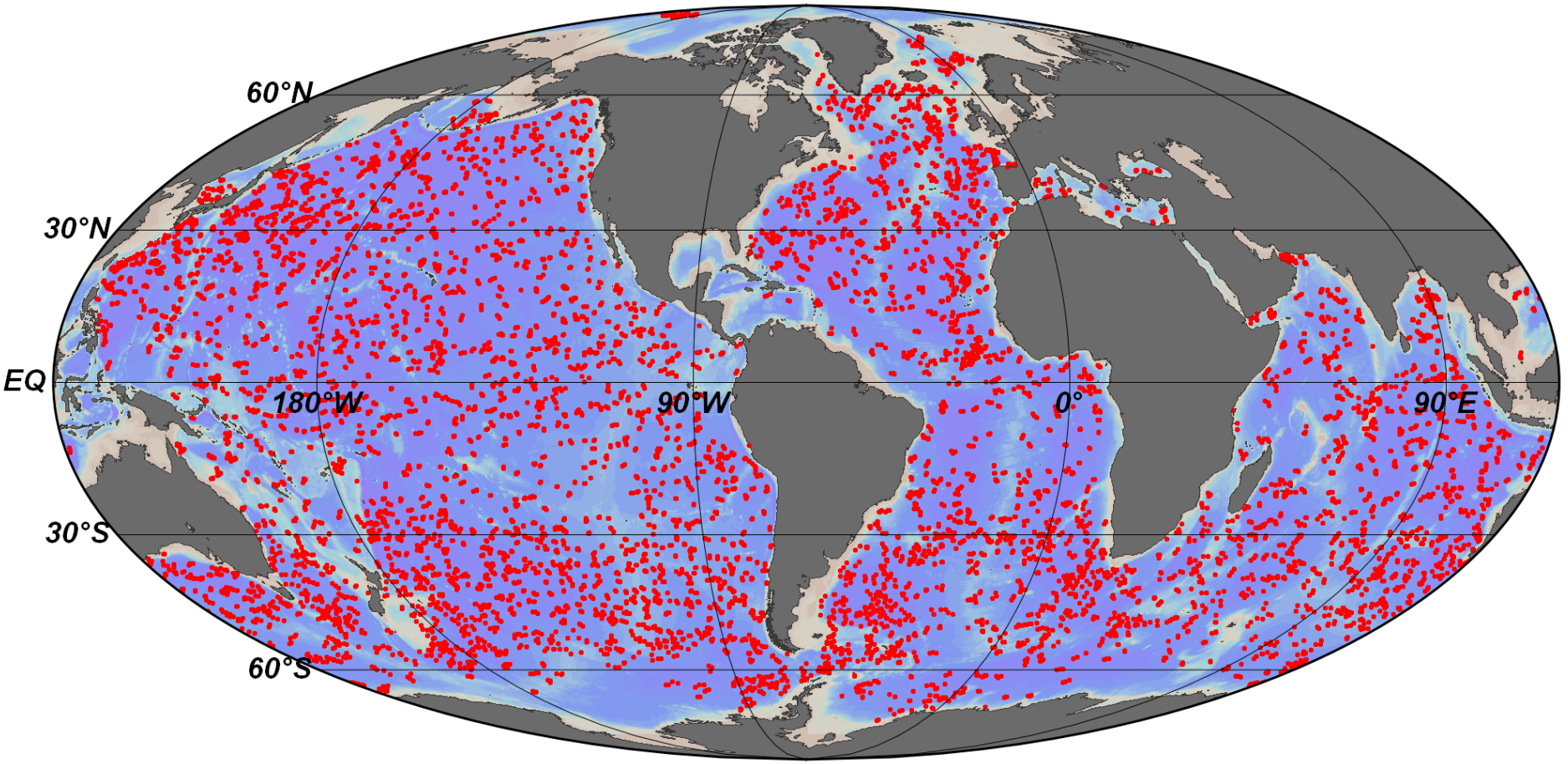
Nov

Karakas etal, 1996

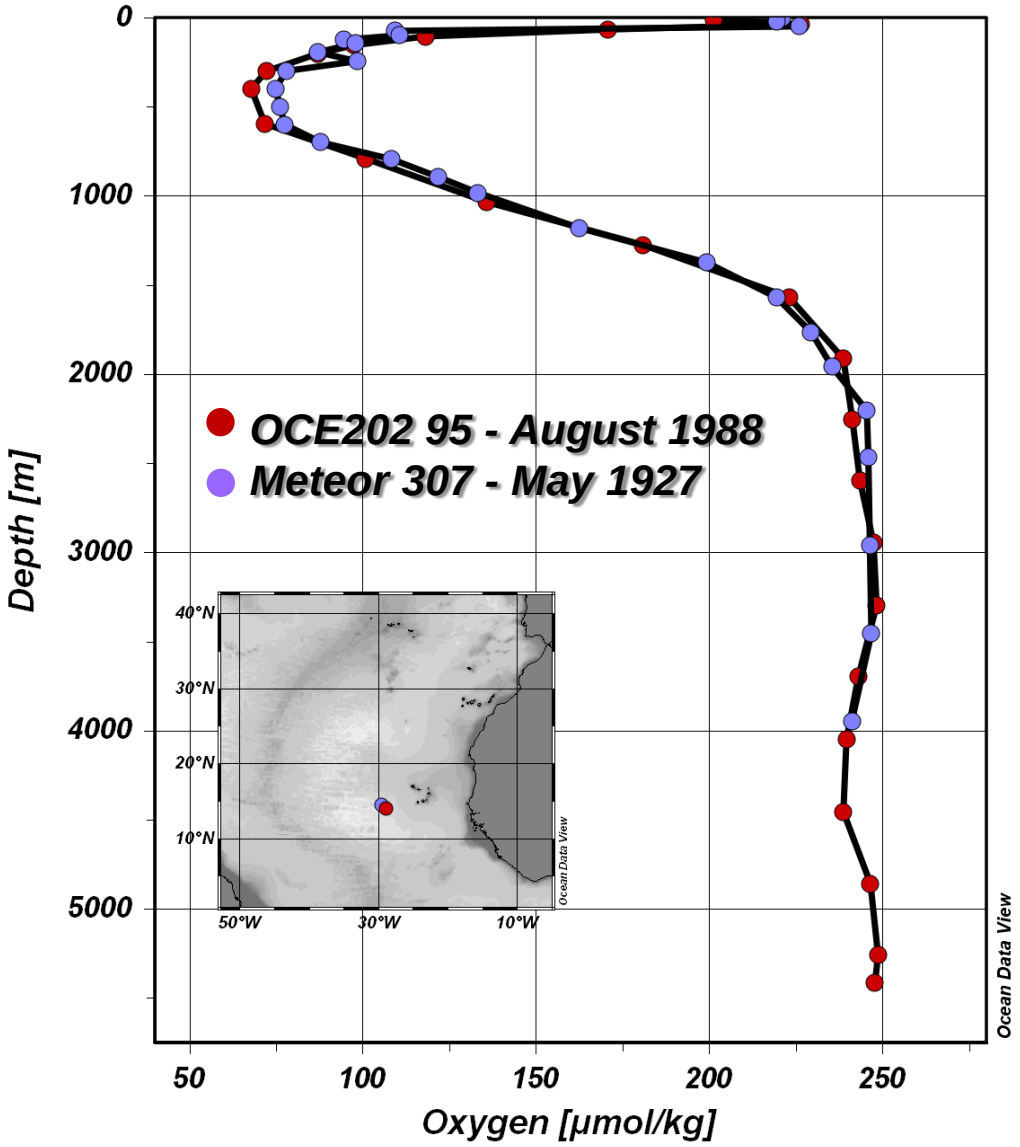
Conclusions: Part 2

- ***Biogenic and lithogenic particle distributions highly variable in space and time.***
- ***Lateral transport is important.***
- ***Observations at any given time far from being representative.***
- ***Optical sensors on moving platforms needed to help improve data coverage.***
- ***Effect on TEI distributions depends on residence time. Fast-turnover TEIs are strongly affected.***

One Month of ARGO Data (Jan/2008 - 10600 profiles)



Integrating nature of ocean tracers. Example Oxygen.



- ***Distribution, downward flux, and composition of biogenic particles are highly variable in space and time.***

- ***Similarity to rain events.***

- ***Variability documented by sediment traps.***

- ***Sediment resuspension requires large bottom shear stresses (large bottom velocities).***
- ***Occurs sporadically under extreme conditions only.***
- ***Local sources and spreading paths highly variable.***